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Gravitational Vacuum Hypothesis and Cosmology with Variable Particle Number

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We discuss a heurisitic model of gravitational vacuum as a set of virtual, radiating planckeons, particles with Planck size (L) and mass ($\hbar/cL$). Combined with Dirac's large number hypothesis, this gives the minimum universe scale factor value $a_{\min} \sim 10^{-13}$ cm, the strong interaction length ($a = L$ just when $a = a_{\min}$). Taking this state as an initial one using standard quantum techniques, we consider particle creation by planckeons. Under some reasonable assumptions we obtain the present number of particles with nucleon mass close to observations, $N \sim 10^{80}$. A criterion for gravitational stability of particles is formulated and some applications of the corresponding mass formula are considered. In particular, Fermi's weak interaction constant is expressed in terms of a and L and a finite value for the neutrino mass is obtained.

1. INTRODUCTION

At present there does not exist a unified theory which could describe satisfactorily all types of physical interactions, gravitational, electromagnetic, weak and strong. Therefore it is of interest to consider model schemes which could lead to important relations connecting the universe and particle characteristics.

One such scheme is described in the present paper, namely, a heuristic model of the gravitational vacuum as a system of radiating planckeons. This hypothesis leads to a relation for masses and characteristic lengths of stable particles.

In Section 3 we show that this model is compatible with the well-known hypotheses of physical constant variation (in the atomic system of units it is the Dirac hypothesis of gravitational constant variation).

In Section 4 we try to apply quantum theory of particle creation in cosmology within the frames of the planckeon vacuum conception. We consider particle creation by nearly closed planckeons which are simulated by nonstationary Robertson–Walker microuniverses. Combined with the Dirac hypothesis, this yields a relation for masses and characteristic lengths of nucleons and the Universe. It is shown that its minimum scale, compatible with the Dirac hypothesis and that of gravitational vacuum, is equal to the strong interaction characteristic length. In addition a criterion of "gravitational stability" of particles is formulated.

In Section 5 we consider applications of the planckeon vacuum model. It is shown that the mass formula gives the nucleon parameters if the particle's effective dimension coincides with its Compton length; it gives the electron mass if we assume that its origin is purely electromagnetic (the effective dimension equals the classical radius); it gives the neutrino mass if the effective dimension is taken to be that of weak interaction. It is also shown that the free path length for all the particles considered has the order of the universe curvature radius if the cross sections are calculated from the effective dimensions.

2. PLANCKEONS

It is natural to believe that a theory able to describe the basic properties of space-time and matter should contain at least three fundamental constants, G, c, and $\hbar$. These are, respectively, today's constants of gravitational interaction, special relativity, and quantum theory. These constants or their combinations allow one to determine the basic measurement scales. For such scales one can choose, e.g., the Planck scales of length, mass, and time:

$$L=(2G\hbar/c^3)^{1/2}\sim 10^{-33}\text{ cm},\qquad m_L=(\hbar c/2G)^{1/2}\sim 10^{-5}\text{ g},$$

$$t_L=L/c=(2G\hbar/c^5)^{1/2}\sim 10^{-43}\text{ s}.$$

In the gravitational vacuum conception to be considered here (Staniukovich, 1972) the space-time is filled with a medium consisting of virtual particles with the Planck size and mass. These particles are called planckeons (Staniukovich, 1965, 1966, 1969) or maximons (Markov, 1965). The quantities L and m_L are the minimum possible radius and maximum possible mass of quantum particles as in this case the gravitational radius $r_g=2Gm_L/c^2=L$ and the Compton wavelength $\lambda=\hbar/mc=L$ coincide.

In the spirit of classical theory such particles can be imagined as closed microuniverses and their virtuality may be related to the fact that their mass with respect to the external space (the space where observers are situated) is zero. However, at distances like the Planck length the gravitational field is probably quantized. Therefore, though microuniverses with Planck parameters can be obtained as solutions to gravitational field equations (Bronnikov, Melnikov, and Staniukovich, 1968, 1976; Bronnikov and Melnikov, 1969), such solutions are of limited interest.

In this paper we do not assume the validity of a certain concrete theory of gravity (say, general relativity); nevertheless, we suppose that some relativistic theory incorporating the space-time curvature concept is valid.

3. POSSIBLE FUNDAMENTAL CONSTANTS VARIATION

Let us discuss some considerations leading to the idea of physical constants variation, in the spirit of papers by Melnikov (1976) and Staniukovich (1971).

At the present epoch, as one easily assures, the so-called Mach relation

$$2GM/c^2a=1 \tag{1}$$

holds to the order of magnitude. Here M is the visible universe mass and a is the scale factor. Such a relation emerges also in any relativistic gravity theory (including general relativity) if the mass M is computed as a product of mean observed density $\bar{\rho}$ by the volume, say, π^2a^3 if the closed elliptic cosmological model is chosen. Equation (1) is then a consequence of the relation $R=\kappa\bar{\rho}c^2$ (where R is the curvature and $\kappa=8\pi G/c^4$), that should be valid to the order of magnitude: one should assert that R is of the order $1/a^2$.

Another relation of importance

$$M/a^2=m_n/\lambda_n^2 \tag{2}$$

where m_n and λ_n are the mass and Compton wavelength of a nucleon, can be obtained from the requirement that the gravitational pressure out of the particle (i.e., the cosmological pressure) is equal to that of the particle itself:

$$GM^2/a^4=Gm_n^2/\lambda_n^4$$

This may be regarded as the condition of gravitational stability of a nucleon. The Dirac formula for the particle number, $N_n=M/m_n=(a/\lambda_n)^2$, is now just a consequence of this condition and thus gains a new physical meaning. Using the uncertainty principle in the form $mc\lambda=\hbar$, one can determine the nucleon parameters as

$$\lambda_n = (L^2 a)^{1/3} \sim 10^{-13}\ \text{cm}, \qquad m_n = hc/\lambda_n \sim 10^{-24}\ \text{g} \tag{3}$$

Thus we have very significant relations (1)–(3) between the fundamental constants G, c, h, m_n, λ_n and the cosmological parameters M and a. It is easily assured that these relations are valid at the present epoch.

It is natural to believe (Melnikov, 1976; Staniukovich, 1971) that these relations involving the variable quantity, a, are not valid only just now, accidentally. Thus, evidently, some of the constants should vary. A choice of "reference" constants which are to remain actually constant corresponds in this case to a choice of measurement units. Depending on this choice we arrive at various known hypotheses on constants variation:

(1) $G, c, \hbar = \text{const}$; $M \sim a, m_n \sim \lambda_n^{-1} \sim a^{-1/3}$, a modified Friedmann model with variable particle masses, except the planckeon mass.

(2) $c, \lambda_n, \hbar, m_n = \text{const}$; $G \sim a^{-1}$; $M \sim N_n \sim a^2$, the Dirac hypothesis (Dirac, 1937; see also Dirac, 1974) with decreasing gravity and increasing particle number.

(3) $c, \lambda_n, M = \text{const}$; $G \sim a$; $\hbar \sim m_n \sim a^{-2}$, the model proposed by Staniukovich (1965, 1971).

(4) $c, \lambda_n, \bar{\rho} = \text{const}$; $G \sim a^{-2}$; $M \sim a^3$; $\hbar \sim m_n \sim a$ ($\bar{\rho} \sim M/a^3$ is the mean density), the later model of Staniukovich (1977).

Other choices of constant scales are also possible.

The first model corresponds to the so-called gravitational system of units. In this system the unit of time is based on the periodicity of gravitational phenomena (e.g., planet motion, the ephemeris time). The second hypothesis corresponds to the atomic system of units. These units are related to measurements performed with devices using quantum properties of matter, i.e., to the atomic standards of frequency and time. One should note that Dirac obtained the same variations from similar considerations but using slightly different relations between the particle and the universe parameters.

It is quite probable that all these models are equivalent and their variety just reflects the arbitrariness in the ways of measurement, i.e., in the choice of references for basic physical quantities. One should conclude that confrontation of theoretical predictions with observational and experimental results should be performed with an obligatory analysis of measurement methods.

Hereafter we will use the atomic system of units corresponding to the Dirac model. In this system

$$G = G_0 a_0 / a, \qquad L = L_0 (a_0/a)^{1/2}, \qquad m_L = m_{L0} (a_0/a)^{-1/2} \tag{4}$$

where the index 0 corresponds to modern values of the parameters. Thus in the atomic system of units the Planck parameters, coinciding with those of gravitational vacuum "cells," change with cosmological expansion.

In the further calculations we put the "true" constants c and $\hbar$ equal 1.

4. GRAVITATIONAL VACUUM AND MATTER CREATION

The well-known calculations concerning cosmological matter creation (see, e.g., Grib et al., 1979, 1980) lead to the conclusion that only a very small fraction of all the particles of the universe could be created from vacuum. We shall try to show that within the frames of the gravitational vacuum model all the matter could have been created by virtual planckeons.

This calculation is aimed at just an estimate, therefore we take a classical model for a planckeon. Namely, we shall assume that it is described by the isotropic cosmological metric

$$ds^2 = a_L^2(\eta)\left[d\eta^2 - d\chi^2 - \sin^2\chi\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right)\right] \tag{5}$$

where we take the simplest time dependence of the scale factor a_L, the linear one:

$$a_L = L + \gamma t \tag{6}$$

where $dt = a_L(\eta)d\eta$; γ is a constant factor characterizing perturbation of the planckeon by the external background. We do not consider a concrete mechanism of these perturbations but suppose that they have quantum nature.

Let us use the methods of quantum theory of massive scalar field in an isotropic world according to Grib et al. (1980). Scalar quanta will be considered to be created by a planckeon and to emerge in the external space-time even though planckeons are described by a closed model, because they are not entirely closed due to fluctuations (Staniukovich et al., 1969).

The particle number density within a single planckeon, under the assumption that at $t=0$ there was vacuum, a state without particles, is determined by the expression

$$n_1(\eta) = (\pi^2 a^3)^{-1} \int d\lambda \cdot \lambda^2 |\beta_\lambda(\eta)|^2 \tag{7}$$

where

$$|\beta_\lambda|^2=(E_\lambda-1)/2 \tag{8}$$

$$E_\lambda(\eta)=\omega^{-1}\left[|\dot{u}_\lambda(\eta)|^2+\omega^2(\eta)|u_\lambda(\eta)|^2\right] \tag{9}$$

The function $u_\lambda(\eta)$ obeys the equation

$$\ddot{u}_\lambda+\omega^2(\eta)u_\lambda=0, \qquad \omega^2=\lambda^2+m^2a_L^2(\eta)+1 \tag{10}$$

and the initial data at $t=\eta=0$

$$u_\lambda(0)=1/[\omega(0)]^{1/2}, \qquad \dot{u}_\lambda(0)=i[\omega(0)]^{1/2} \tag{11}$$

If

$$\gamma t \ll L, \qquad \gamma \ll 1, \qquad \gamma \ll Lm \tag{12}$$

(the first condition means smallness of the planckeon excitation, the second one that the expansion is slow enough, and the third one adiabaticity, $H_L \ll m$), then, solving equation (10) with (6) and inserting the solution into (7)–(9), we come to the following estimate of particle number density created by one planckeon for a period of time Δt:

$$n_1(t)=\frac{mH_L^2}{4\pi^2}\int_1^\infty dx\frac{(x^2-1)^{1/2}}{x^5}\cdot\sin^2 xmt\approx\frac{m^3H_L^2\Delta t^2}{4\pi^2}\int_1^\infty dx\frac{(x^2-1)^{1/2}}{x^3}$$

$$\approx m^3H_L^2\Delta t^2/16\pi \tag{13}$$

$$x\overset{\text{def}}{=}\left(1+\lambda^2/m^2a_L^2\right)^{1/2}, \qquad H_L=a_L^{-1}\cdot da_L/dt=\gamma/L$$

The integral is majorated by the substitution $\sin^2 xmt\rightarrow(xmt)^2$ which faintly affects the result.

In one planckeon volume (π^2L^3 according to the elliptic model) for a period Δt there emerges the number of particles

$$\Delta N_1=\pi^2L^3n_1=(\pi/16)m^3H_L^2L^3\Delta t^2 \tag{14}$$

For the whole universe volume [using the elliptic closed model and assuming that the planckeons fill the universe densely, i.e., the number of planckeons is $(a/L)^3$], taking into account that $H_L=\gamma/L$, we obtain for $\Delta t=L$ the particle number gain

$$\Delta N = (\pi/16) m^3 H_L^2 \Delta t^2 a^3 = (\pi/16) m^3 \gamma^2 a^3 \tag{15}$$

We shall assume that mean time of existence for a virtual planckeon is $\Delta t = L$. Expression (13) gives the particle number gain expectation value related to a single planckeon; in fact, $\Delta N_1 \ll 1$, i.e., a very small fraction of the planckeons can create a particle during the time of existence. At the next period of time $\Delta t = L$ new planckeons emerge, and naturally within them particles are created from vacuum again. Thus the particle number gains within the universe are added. Hence, taking into account that Δt is extremely small compared with the universe age, one can write

$$\frac{\Delta N}{\Delta t} \approx \frac{dN}{dt} = \frac{\pi}{16L} m^3 \gamma^2 a^3 = \frac{\pi}{16} \frac{m^3 \gamma^2}{L_0 a_0^{1/2}} a^{7/2} \tag{16}$$

[In the last equality we insert the dependence $L(a)$ according to the Dirac model.] The dependence of the excitation factor γ on the scale factor a or the universal time remains so far unknown. To find it out let us take into account that according to Dirac's large number hypothesis (valid in the system of units chosen)

$$N = N_0 (a/a_0)^2 \tag{17}$$

where N_0 is the modern particle number. Thus the factor γ can be determined from the relation

$$N_0 \frac{da}{dt} = \frac{\pi m^3 a_0^{3/2}}{32 L_0} \gamma^2 a^{5/2} \tag{18}$$

if the dependence $a(t)$ is known. Taking for simplicity $a = t$ (noting that just the linear law of expansion is compatible with Dirac's hypothesis), we get

$$\gamma = \left(\frac{32 L_0 N_0}{\pi m^3 a_0^{3/2}} \right)^{1/2} a^{-5/4} \tag{19}$$

Let us consider the initial conditions of the model evolution. It is quite clear that the hypothesis in which planckeons fill the space densely requires that the universe should contain at least one "cell" with the Planck dimensions. Thus the minimum universe dimension $a_{\min}$ is determined from the condition $N_{\text{cells}}=(a/L)^3=1$, $a=L$, whence

$$a_{\min}=\left(L_0^2 a_0\right)^{1/3}\approx 10^{-13}\text{ cm} \tag{20}$$

Note that this condition corresponds just to the principal lower bound of classic description in cosmology: the scale factor equals the Planck length.

On the other hand, $a_{\min}$ coincides with the nucleon Compton wavelength λ_n [see (3)]. As at any epoch particle masses obey the principal limitation $m_L(a)>m$, it is clear that, at least at very early times, particles no heavier than nucleons can be created.

Let us return to relation (17) and put $N=1$ for $a=a_{\min}$. This reflects the fact of creation of the first nucleon, which at that moment coincides with the universe itself. Thus we obtain

$$N_0=(a_0/a_{\min})^2 \tag{21}$$

Comparing (20) and (21), we arrive at the result

$$N_0=(a_0/L_0)^{4/3}\approx 10^{81} \qquad \left(a_0\approx 10^{28}\text{ cm}\right) \tag{22}$$

coinciding with the well-known estimates for nucleon number in the universe. Now, with $m=m_n$, (19) gives

$$\gamma^2=(32/\pi)\left(L_0^2 a_0\right)^{5/6}a^{-5/2}=(32/\pi)(\lambda_n/a)^{5/2} \tag{23}$$

For the present epoch, $a=a_0$,

$$\gamma^2=(32/\pi)(\lambda_n/a_0)^{5/2}\sim 10^{-102} \tag{24}$$

i.e., at present planckeon excitation is extremely small. For $a=a_{\min}$ we get

$$\gamma=(32/\pi)^{1/2}\sim 1 \tag{25}$$

Conditions (12) for $a=a_0$ are reduced to $\gamma\ll 10^{-20}$ and for $a=a_{\min}$ to $\gamma\ll 1$. One can conclude that the approximation adopted is valid for $a\gg a_{\min}$, i.e., up to the earliest epochs.

It should be stressed that these estimates are meaningful if the real particles' maximum mass has the order m_n. Because of strong dependence on m in the particle creation formulas ($\sim m^3$), presumably particles with maximum possible mass will be created. Values of m at each epoch are limited by the value $m_L(a)\sim a^{1/2}$. Thus if the real particles' mass spectrum contains values $m_i \gg m_n$, then creation of such particles is "switched on" only at the moment when $m_i^{-1}=L(a)$, i.e., $a=a_0/(\mathrm{m}L_0)^2$. When created, such particles should decay into stable ones, mainly nucleons.

Using the Mach relation, one can transform the obtained formula $a_{\min}=\lambda_n=(L_0^2 a_0)^{1/3}=(L^2 a)^{1/3}$ to the form (2). Indeed, from $\lambda^3=L^2 a=2Ga$ it follows

$$m/\lambda^2=1/2Ga=M/a^2=m_L/aL \tag{26}$$

Let us assume that this relation is applicable to any stable particles, i.e., that the following gravitational stability criterion is fulfilled: for a stable particle, local gravitational pressure (energy density) is equal to the universe pressure at some characteristic interaction radius l_i. Thus

$$M/a^2=m_n/\lambda_n^2=m_e/l_e^2=m_\nu/l_\nu^2=m_i/l_i^2, \qquad \text{or } L^2 a=\lambda_i l_i^2 \tag{27}$$

As we shall see further, for an electron l_e is its classical radius, for a neutrino l_ν is the weak interaction characteristic length, etc.

5. APPLICATIONS

5.1. As has been already pointed out, equation (27) leads to the nucleon parameters [see also (3)]: the strong interaction characteristic length is

$$\lambda_n=(L^2 a)^{1/3}=1/m_n$$

It should be stressed that l coincides with the Compton wavelength only for hadrons, i.e., nucleons, or, even more precisely, π mesons.

5.2. The Electron Mass and the Fine Structure Constant. Suppose that the electron mass is entirely of electromagnetic origin. Then $m_e c^2=e^2/l_e$ and (27) give

$$m_e/l_e^2=1/2Ga=1/2G_0 a_0$$

Excluding l_e, it is easy to obtain

$$m_e/m_L=\alpha^{2/3}(L/a)^{1/3}$$

where $\alpha=e^2$ is the fine structure constant. Thus

$$m_e=(\alpha^{2/3}/L)(L/a)^{1/3}\approx 10^{-27}\text{ g}$$

in good agreement with the experiment. A viewpoint that α is possibly related to the maximum (a) and minimum (L) dimensions of the universe, $\alpha^{-1}=\ln(a/L)\approx 137$, is stated by Staniukovich (1971).

5.3. Neutrino Mass. Consider a mass whose Compton wavelength is $(aL)^{1/2}=l_{\text{pl}}$ [the effective size of a planckeon by (27), i.e., l_i for which $m_i=m_L$). Evidently this mass is $m_\nu=(aL)^{-1/2}$; the corresponding effective dimension l_ν is found from (27), viz., $m_\nu/l_\nu^2=1/2Ga$. Thus we obtain

$$l=l_\nu=L(a/L)^{1/4}, \qquad l_\nu\approx 0.3.10^{-17}\text{ cm} \qquad \text{for } a=a_0=10^{28}\text{ cm}$$

This value is close to the characteristic length of weak interactions. Hence Gamov's empirical formula $l_\nu=G_F^{1/2}$ enables us to express Fermi's weak interaction coupling constant in terms of the fundamental quantities:

$$G_F=a^{1/2}L^{3/2}$$

The value of $m_\nu=(aL)^{-1/2}$ ($\approx 0.7.10^{-35}$ g$\approx 4.10^{-3}$ eV for $a=a_0$) is in agreement with astrophysical limitations for the neutrino mass ($m_\nu<40$ eV by Dolgov and Zeldovich, 1980) and is not very far from recent laboratory data (Liubimov et al., 1980).

Note that in this approach G_F and m_ν are, unlike the strong and electromagnetic interaction characteristics, functions of the scale factor a and change with cosmic time:

$$G_F=(aL^2)^{3/4}a^{-1/4}=a_0^{1/2}L_0^{3/2}(a/a_0)^{-1/4},$$

$$m_\nu=(aL^2)^{-1/4}a-^{1/4}=a_0^{-1/2}L_0^{-1/2}(a/a_0)^{-1/4}$$

5.4. Free Path Length. For a gas particle the free path length is known to be

$$\bar{\lambda}=\lambda_{\text{free path}}=(\sigma n)^{-1}$$

where σ is the interaction cross section assumed to be close to l^2 and n is the particle number density. If we assert that the total mass M contains only one sort of particle, i.e., $n\approx M/m\pi^2a^3$, then we get $\bar{\lambda}\gtrsim a$, so the matter behaves like a perfect gas.

A similar analysis in the gravitational system of units is given in the papers by Staniukovich (1980) and Staniukovich and Melnikov (1980).

REFERENCES

Bronnikov, K. A., and Melnikov, V. N. (1969). Preprint ITF-69-21, Kiev.

Bronnikov, K. A., Melnikov, V. N., and Staniukovich, K. P. (1968). Preprint ITF-68-69, Kiev.

Bronnikov, K. A., Melnikov, V. N., and Staniukovich, K. P. (1976). In *Trudy GR-5*. Tbilisi University Press, Tbilisi.

Dirac, P. A. M. (1937). *Nature*, **139**, 323.

Dirac, P. A. M. (1974). *Proceedings of the Royal Society of London Series A*, **338**, 439.

Dolgov, A. D., and Zeldovich, Ya. B. (1980). *Uspekhi Fizicheskikh Nauk*, **130**, 559.

Grib, A. A., Mamayev, S. G., and Mostepanenko, V. M. (1980). *Kvantovye Effekty v Intensivnykh Vneshnikh Poliakh (Quantum Effects in Intensive External Fields*, in Russian). Atomizdat, Moscow.

Liubimov, V. A., et al. (1980). Preprint ITEF No. 62, Moscow.

Markov, M.A. (1965). *Supplement to Progress in Theoretical Physics*, Extra Number.

Melnikov, V. N. (1976). In *Problemy Teorii Gravitatsii i Elementarnykh Chastits (Problems in Gravitation Theory and Particle Theory*, in Russian)(*PTGEC*), Vol. 7. Atomizdat, Moscow.

Staniukovich, K. P. (1965). *Gravitatsionnoe Pole i Elementarnye Chastitsy (Gravitational Field and Elementary Particles*, in Russian). Nauka, Moscow.

Staniukovich, K. P. (1966). In *PTGEC*, Vol. 1.

Staniukovich, K. P. (1971). In *Teoriya Otnositelnosti i Gravitatsiya (Relativity and Gravitation*, in Russian). MOIP, Moscow.

Staniukovich, K. P. (1972). In *Trudy VNIIFTRI*, Vol. 16 (46). Moscow.

Staniukovich, K. P. (1977). In *PTGEC*, Vol. 8.

Staniukovich, K. P. (1980). In *GR-9 Abstracts*. Jena.

Staniukovich, K. P., et al. (1969). In *PTGEC*, Vol. 2.

Staniukovich, K. P., and Melnikov, V. N. (1980). In *Goreniye i Vzryv v Kosmose i na Zemle (Burning and Explosion in Space and on the Earth*, in Russian). VAGO AN SSSR, Moscow.